Finding Errors of Hybrid Systems by Optimising an Abstraction-Based Quality Estimate

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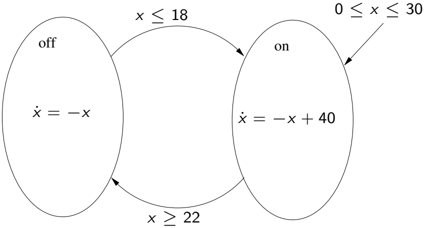
July 3, 2009

Hybrid Systems

Thermostat:



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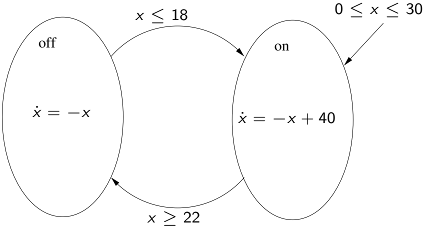

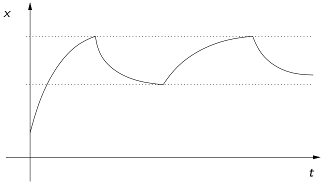

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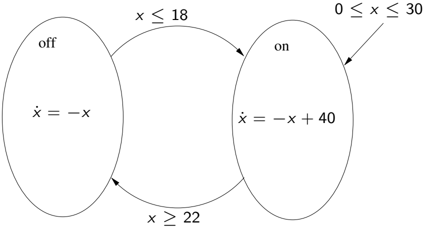

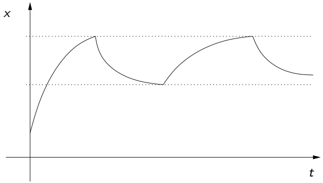

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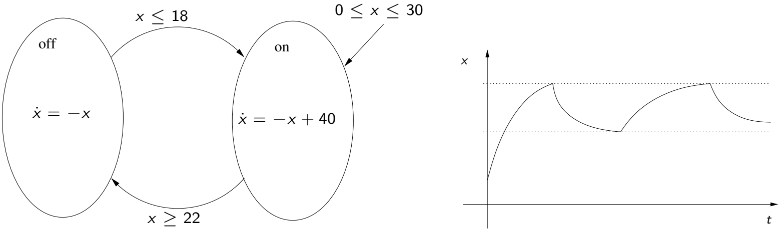

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Also continuous state can jump discontinuously (state updates)

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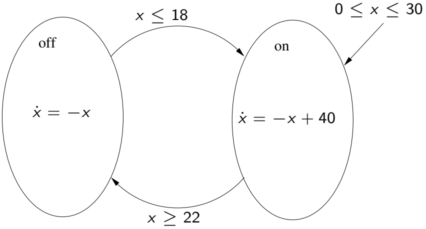

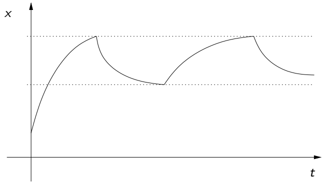

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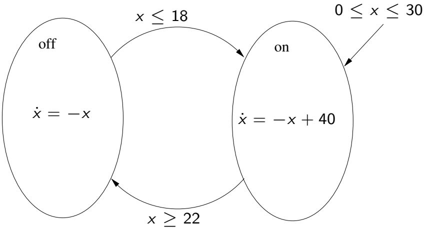

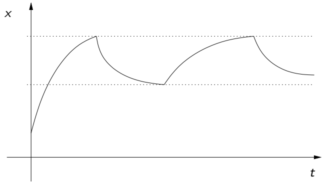

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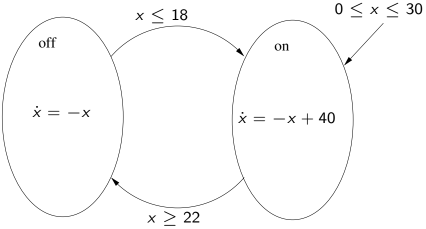

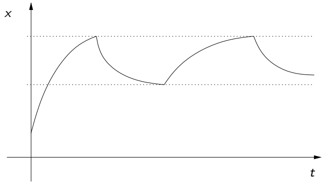

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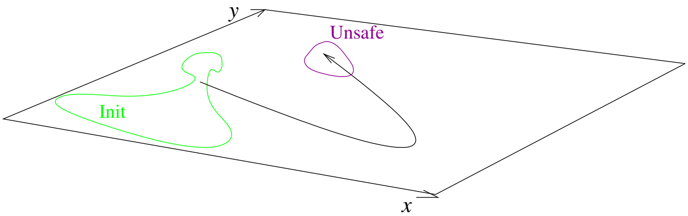
Motivation: embedded systems, motor gears, ...

System Correctness

Error trajectory: trajectory from initial to unsafe state



System is *correct (safe)* if it does not contain an error trajectory

Problem Definition

Observation:

- for ordinary differential equations forward reachability computation (as used in most verification algorithms) only with over-approximation.
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So: design algorithm to detect incorrectness (*falsification* algorithm)

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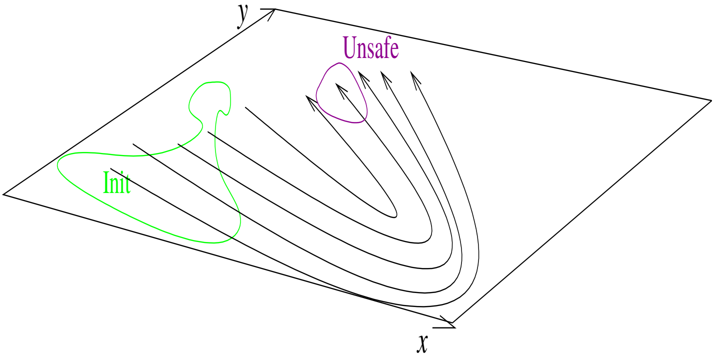
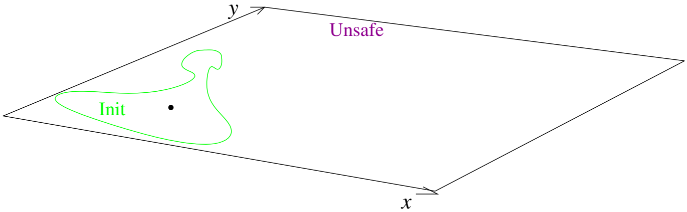


So, problem: finding a startpoint of an error trajectory.

Computing trajectories

simulation



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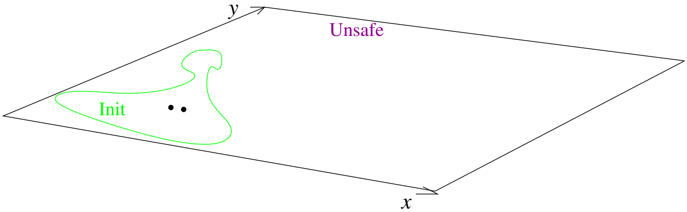

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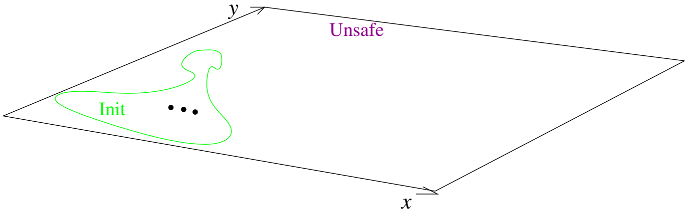

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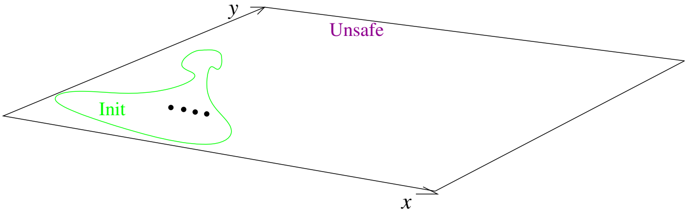

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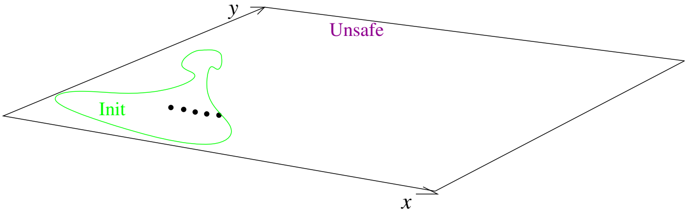

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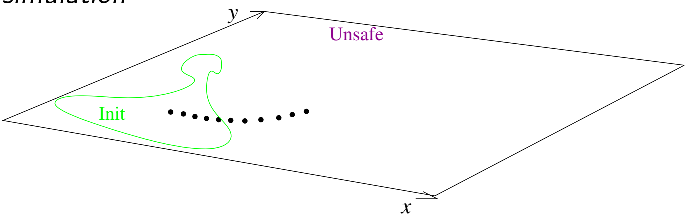
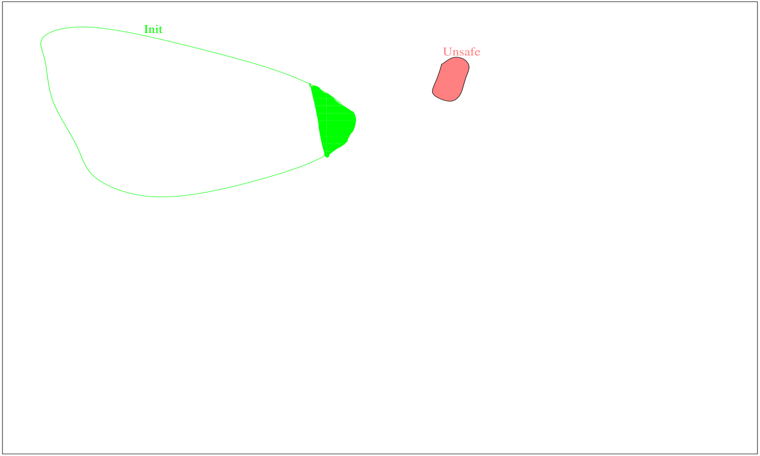


Naïve Method

starting points of simulations



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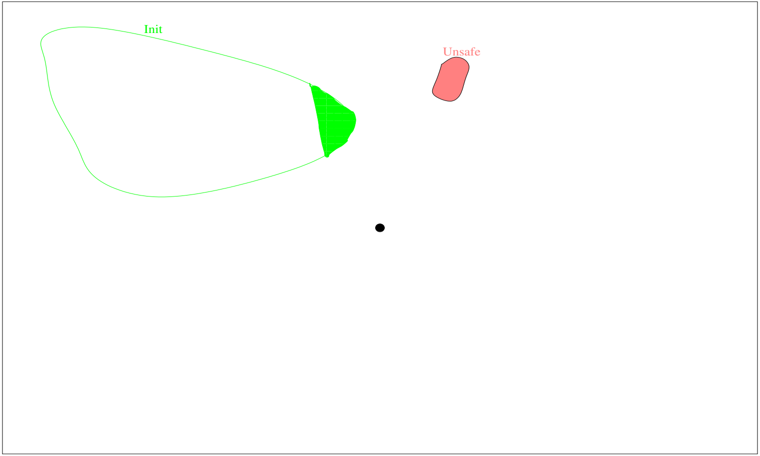

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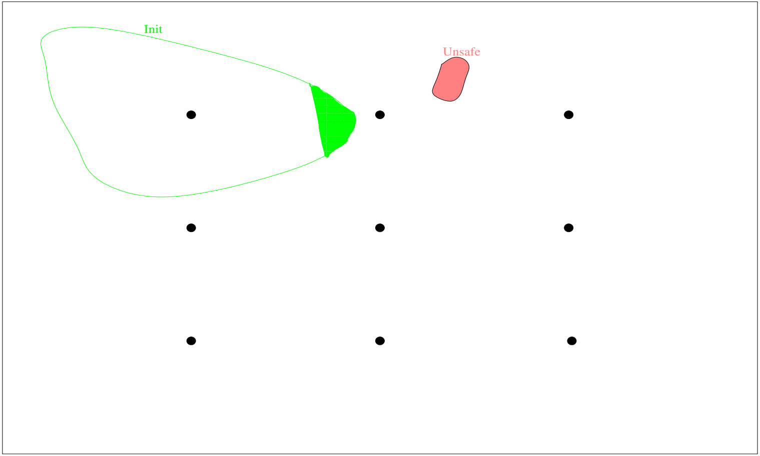

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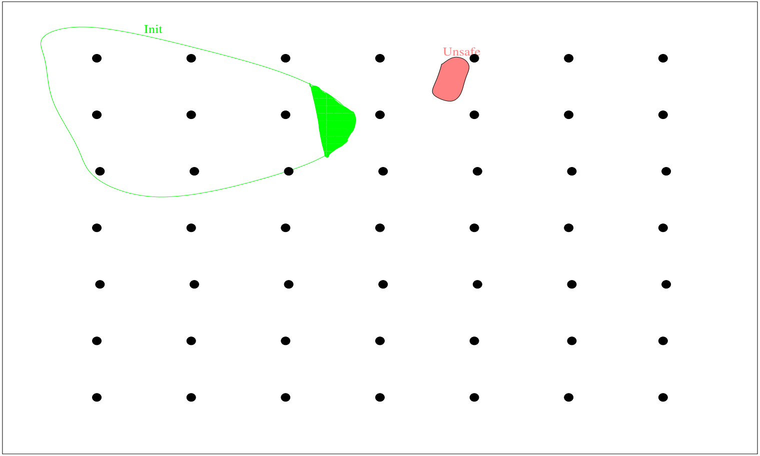

What to Do about the Naïve Method?

Naïve because:

- It runs forever on safe systems.
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Therefore we will . . .

- . . . alternate verification and falsification cycles;
- . . . prefer the more promising simulations;
- . . . cancel simulations when they do not look promising anymore.

HSolver Abstraction

Verification tool: HSolver
(`http://hsolver.sourceforge.net/`)

HSOLVER Abstraction

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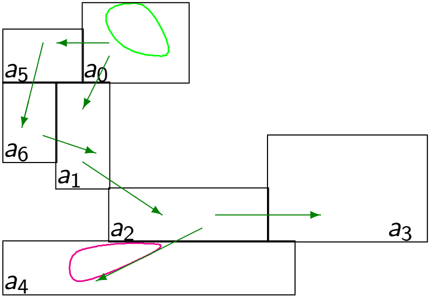
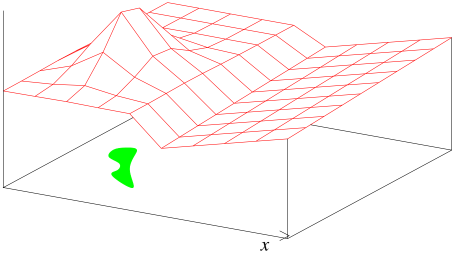
- The statespace is partitioned into finitely many boxes.
- Interval arithmetic is used to compute the abstract transitions.
- Overapproximation used for verification.



- If necessary, the abstraction is refined by splitting a box.
- State space pruning

Our Method: Main Idea



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Optimise the quality estimate.

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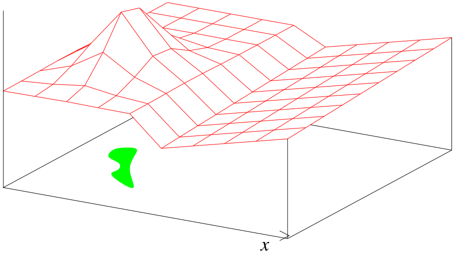

Use real-valued *quality estimate* to approximate "given point is close to an initial point of an error trajectory".

Optimise the quality estimate.

- How to define this function?
- How to find the optimum?

Defining the Quality Estimate

Overall approach:

- Start a simulation
- Compute closeness to error trajectory on the fly
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Solution: Use information from abstraction, s.t. fine enough abstraction will result in reliable quality estimate

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Note: leaving abstraction means "no error trajectory"

But: there might still be an error trajectory nearby

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Maximal closeness of any individual simulation point

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This is not
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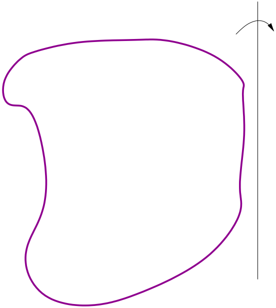

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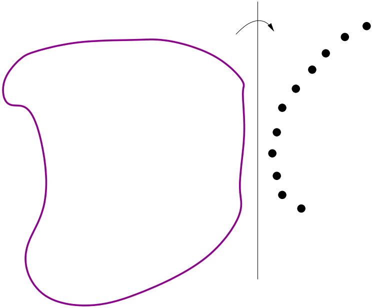

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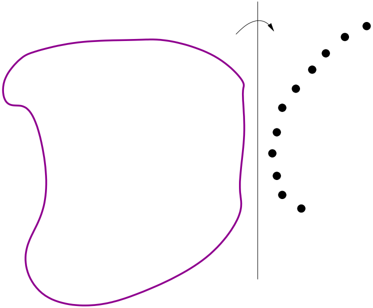
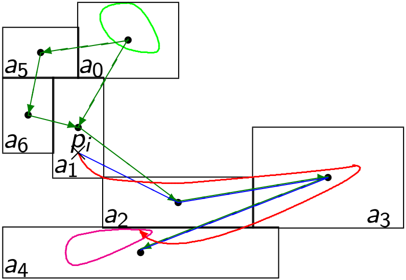


Measure closeness using
abstraction



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Hope: abstraction eventually good enough for reliable strategy

Optimising the Quality Estimate

From boxes that might contain initial states,
start numerical local optimisation.

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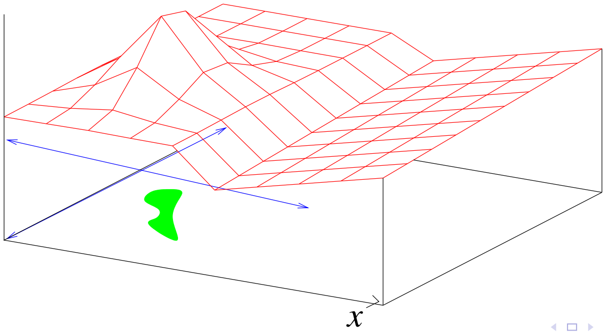

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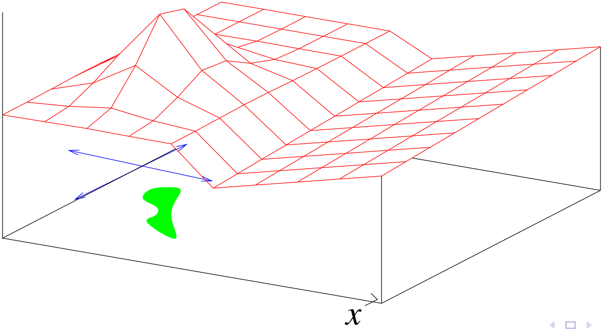

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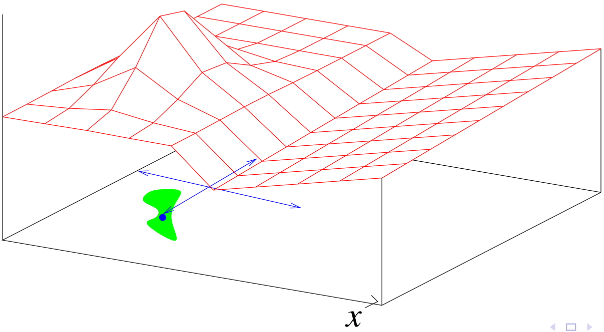

Experiments

	our algorithm				naïve algorithm	
Example	*sim_cnc*	time	ref.	sim.	time	sim.
convoi	200	0.04	0	1	∞	∞
eco	400	0.1	0	1	0.1	1
eco	200	2.1	10	63	0.1	1
focus	200	0.1	0	10	0.04	1
focus	20	29.7	434	288	0.04	1
parabola	105	0.0	0	1	∞	∞
parabola	30	18.0	353	113	∞	∞

Jumps

- For simplicity, we did not explain here how the quality estimate is defined in the presence of jumps.
- Our current implementation did find error trajectories with up to 2 (necessary) jumps.
- Encouraging first result, but topic for future work.

Conclusion

Main Observations:

- Local search can help to find error trajectories.
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Future work:

- More mathematical intelligence (e.g., derivatives)
- Reasoning forward and backward
- Non-deterministic evolution