Nitpick:
A counterexample generator for higher-order logic based on a relational model finder

Jasmin Blanchette & Tobias Nipkow
TU München

Isabelle/HOL

small-kernel interactive theorem prover
for higher-order logic

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Quickcheck

random generation of variable assignments

+ fast − requires exec.

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Refute

finite model finding using SAT

+ general − slow

Nitpick

finite model finding using Kodkod

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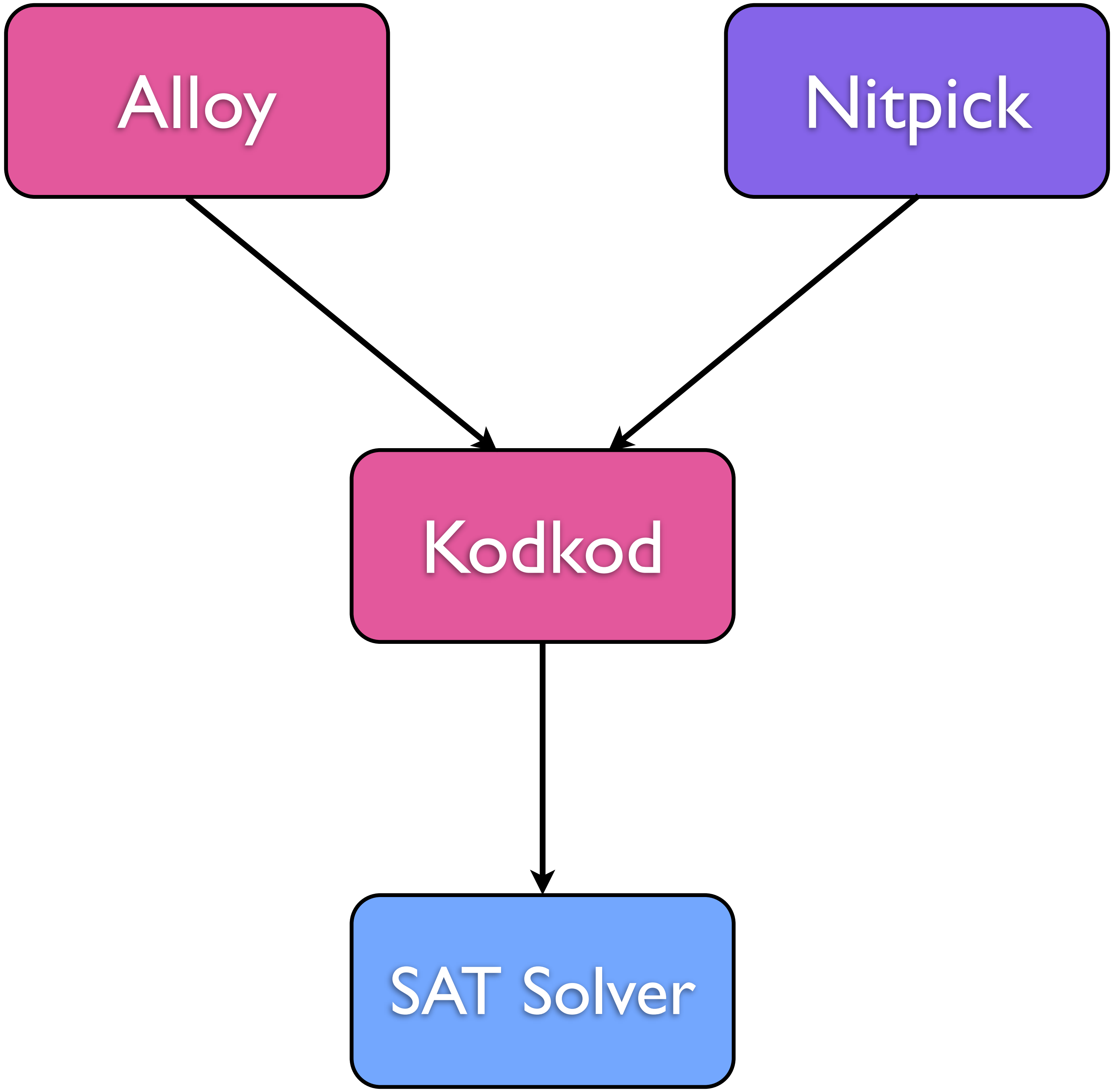
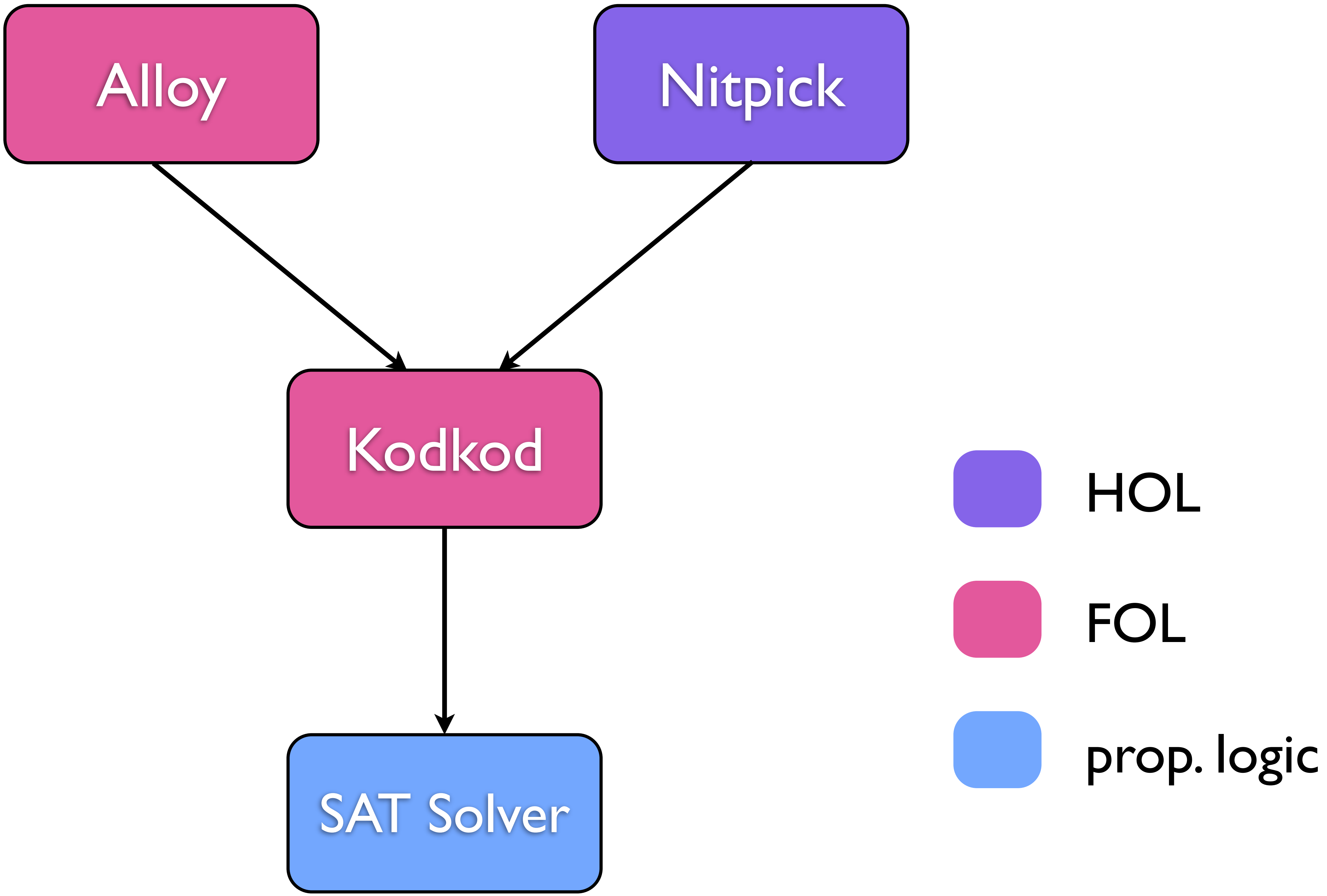
Kodkod

finite model finder for FOL w/ relational calc.

based on SAT

backend of the Alloy Analyzer


Alloy
Nitpick
Kodkod
SAT Solver


Alloy
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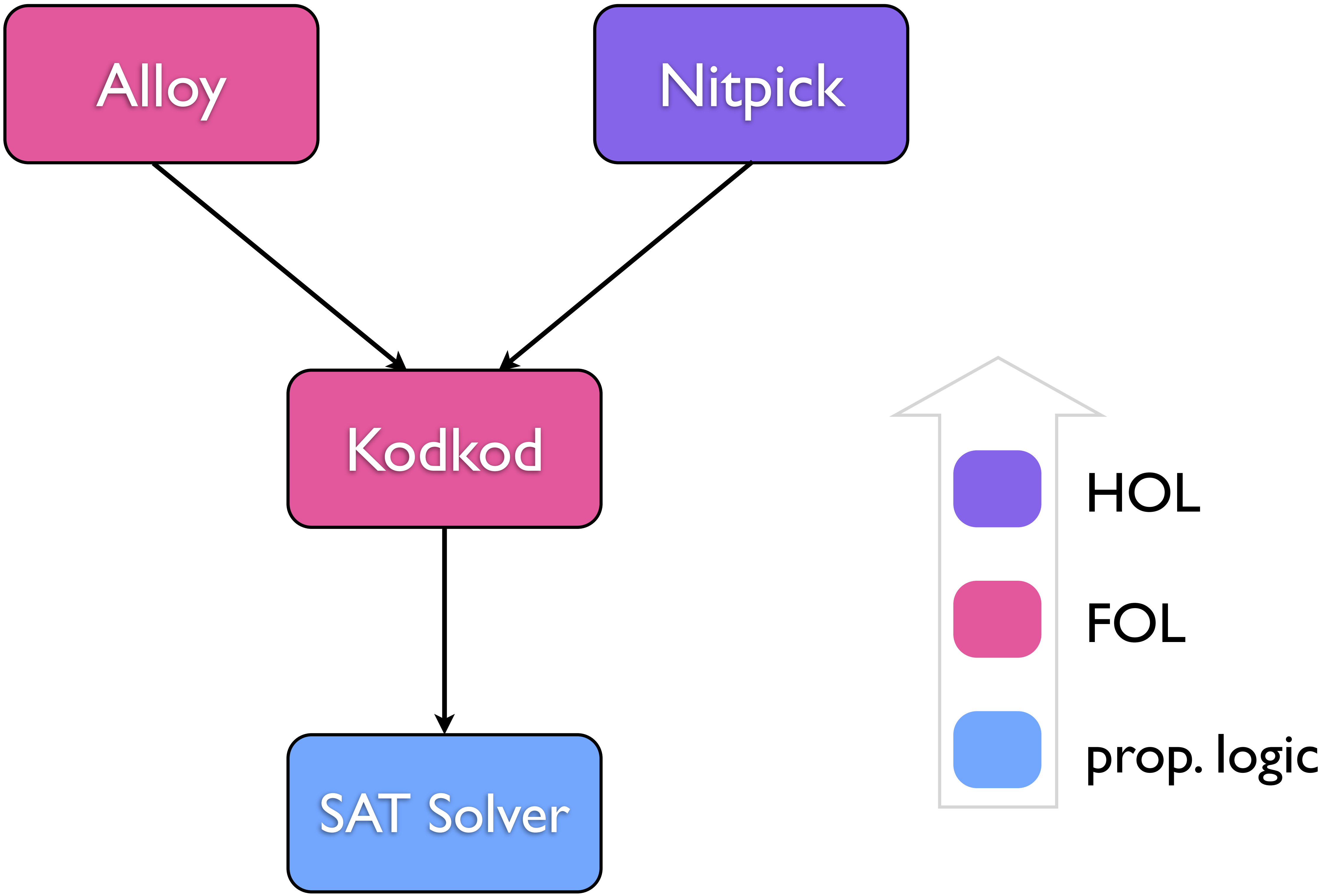
SAT Solver
HOL
FOL
prop. logic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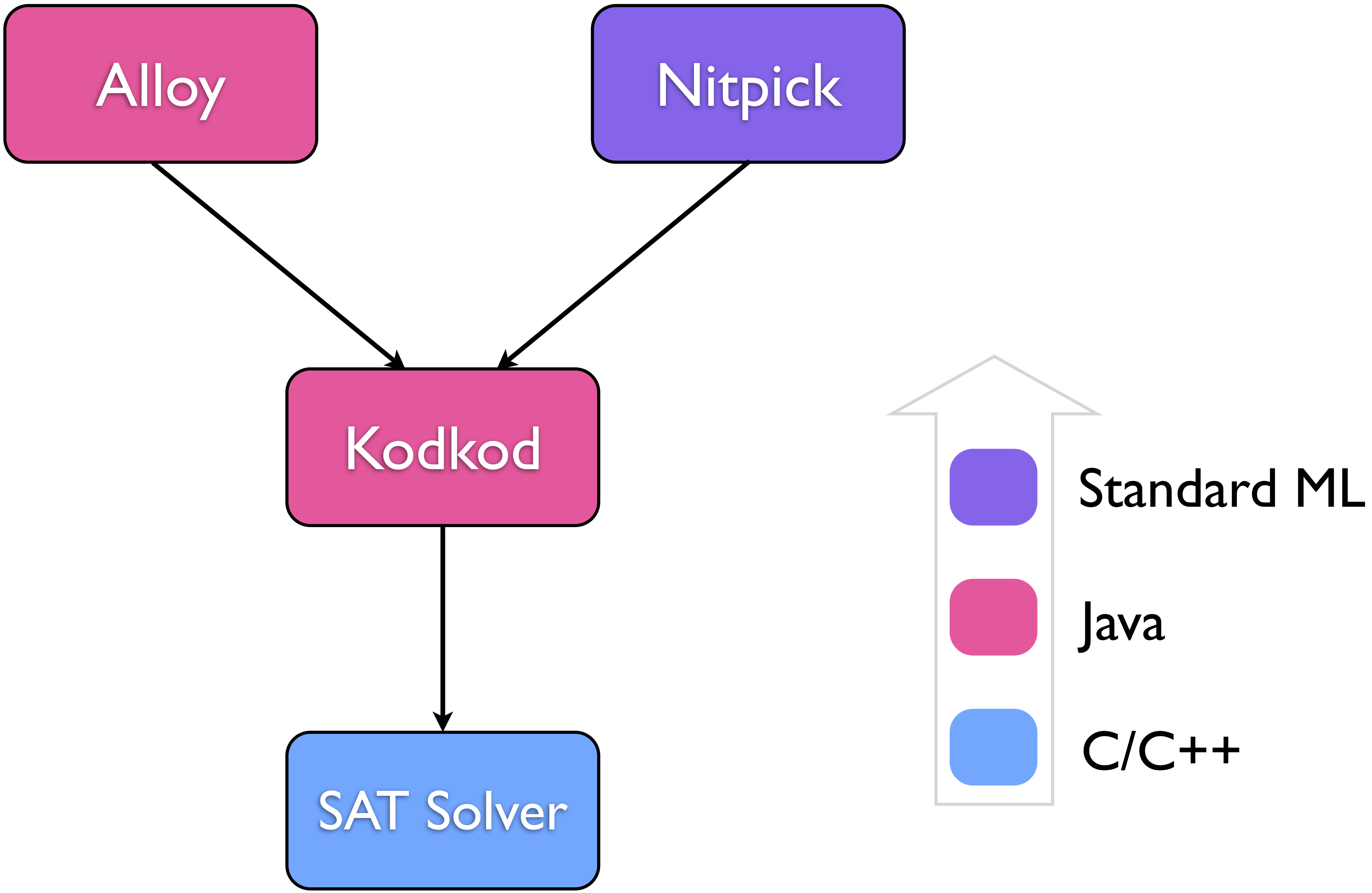
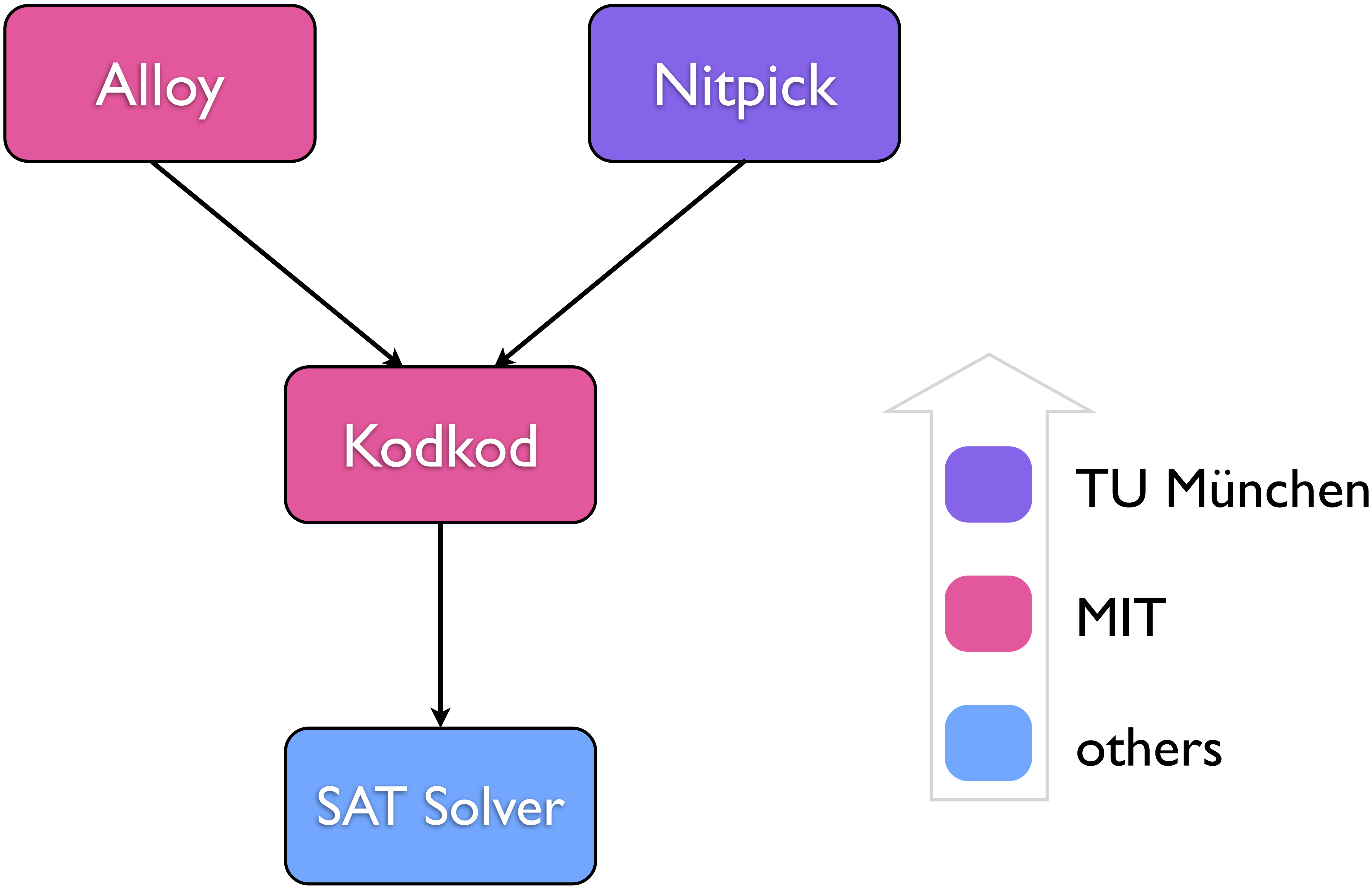

Alloy
Nitpick
Kodkod
SAT Solver
Standard ML
Java
C/C++



Alloy
Nitpick
Kodkod
SAT Solver
TU München
MIT
others

Nitpick in a nutshell

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scope enumeration

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numbers,
datatypes,
ind. predicates,
rec. functions

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- λ-abstractions ➙ set comprehensions

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- Kleene logic
- potential counterexamples

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- recursive functions: defined by their eq. spec.

Example: α *list*

scope: $|\alpha| = 2,\ |\alpha\ \mathit{list}| = 3$

ctors: $\mathit{Nil}^{\alpha\ \mathit{list}}$

$\mathit{Cons}^{\alpha \to \alpha\ \mathit{list} \to \alpha\ \mathit{list}}$

universes: $\{[], [a_1], [a_2]\}$

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{ lexicographic_order
sizechange

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unrolled eq.: $even_{\mathbf{0}}(n) = \bot$

$even_{\mathbf{k+1}}(n) =$

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Function specialization

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$$\mathit{map}'\ [\,]\ =\ [\,]$$
$$\mathit{map}'\ (x \cdot \mathit{xs})\ =\ (x + 2) \cdot \mathit{map}'\ \mathit{xs}$$

$$\mathit{map}'\ \mathit{ks}$$

Evaluation

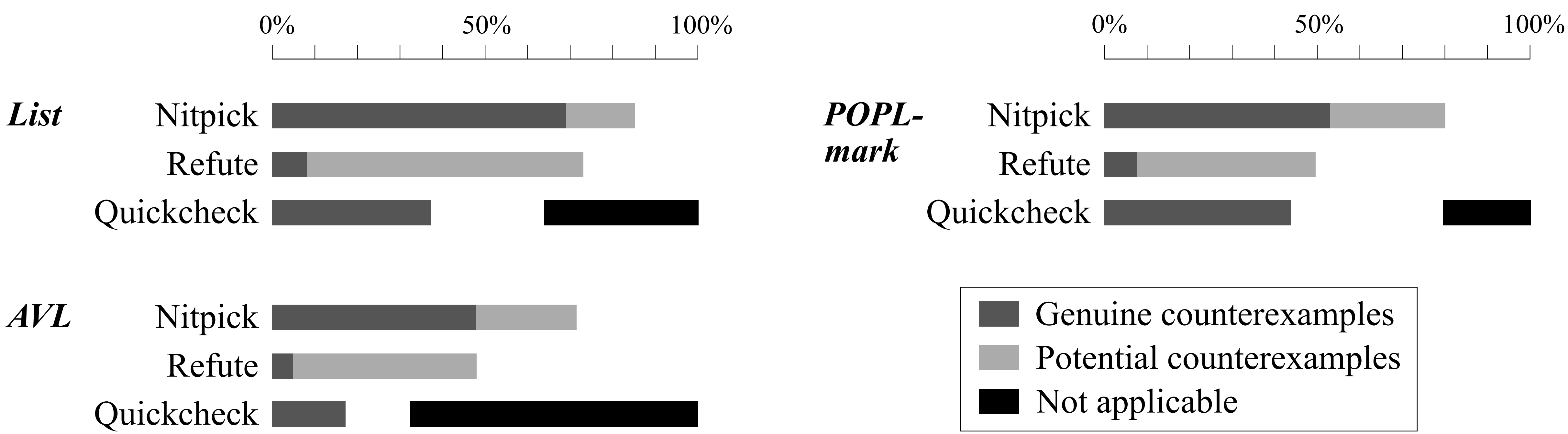

Fig. 1. Success rates of the counterexample generators on three theories

Status

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- a few users + TPTP
- found two bugs in TPS prover!

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Future work

- more optimizations (speed and precision)
- evaluations